Math 10B with Professor Stankova
Quiz 12; Tuesday, 4/17/2018
Section #203; Time: 9:30 AM
GSI name: Roy Zhao **Name:** ______________________

Circle True or False or leave blank. (1 point for correct answer, −1 for incorrect answer, 0 if left blank)

1. **TRUE** False If two vectors are perpendicular to each other (they form an angle of 90°), then their dot product is 0.

> **Solution:** The dot product is $\vec{v} \circ \vec{w} = |\vec{v}||\vec{w}|\cos(\alpha)$ but $\alpha = 90°$ and $\cos\alpha = 0$ so the dot product is 0.

2. **TRUE** False If we have found two different solutions to $A\vec{x} = \vec{b}$, then $\det(A) = 0$.

> **Solution:** If we have found two different solutions, then we know that there must be infinitely many solutions so $\det(A) = 0$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) Let $A = \begin{pmatrix} 2 & 1 & 4 \\ 0 & 2 & 1 \\ -1 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

   (a) (2 points) Calculate $B\vec{v}$.

> **Solution:**
> $$B\vec{v} = \begin{pmatrix} 13 \\ 5 \end{pmatrix}$$

   (b) (4 points) Find a solution to $B\begin{pmatrix} x \\ y \end{pmatrix} = \vec{v}$.

> **Solution:** To solve $B\vec{x} = \vec{v}$, we multiply by B^{-1} to get
> $$\vec{x} = B^{-1}\vec{v} = \frac{1}{3 \cdot 2 - 5 \cdot 1}\begin{pmatrix} 2 & -5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -8 \\ 3 \end{pmatrix}$$

(c) (1 point) Is it unique? Why?

Solution: It is unique because $\det(B) \neq 0$.

(d) (3 points) Calculate $\det(A)$.

Solution: We can calculate it as $2 \cdot 2 \cdot 0 + 1 \cdot 1 \cdot (-1) + 4 \cdot 0 \cdot 1 - 2 \cdot 1 \cdot 1 - 1 \cdot 0 \cdot 0 - 4 \cdot 2 \cdot (-1) = 5$.